3.3.3 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 3 materials.

In Exercises 1 - 10, for the given polynomial:

- Use Cauchy's Bound to find an interval containing all of the real zeros.
- Use the Rational Zeros Theorem to make a list of possible rational zeros.
- Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.

For help with these exercises, click one or more of the resources below:

- Using the Rational Zeros Theorem
- Using Descartes' Rule of Signs

1. $f(x) = x^3 - 2x^2 - 5x + 6$
2. $f(x) = x^4 + 2x^3 - 12x^2 - 40x - 32$
3. $f(x) = x^4 - 9x^2 - 4x + 12$
4. $f(x) = x^3 + 4x^2 - 11x + 6$
5. $f(x) = x^3 - 7x^2 + x - 7$
6. $f(x) = -2x^3 + 19x^2 - 49x + 20$
7. $f(x) = -17x^3 + 5x^2 + 34x - 10$
8. $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$
9. $f(x) = 3x^3 + 3x^2 - 11x - 10$
10. $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

In Exercises 11 - 30, find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.

For help with these exercises, click on the resources below:

- Using the Rational Zeros Theorem and Synthetic Division to Factor Polynomial Functions

11. $f(x) = x^3 - 2x^2 - 5x + 6$
12. $f(x) = x^4 + 2x^3 - 12x^2 - 40x - 32$
13. $f(x) = x^4 - 9x^2 - 4x + 12$
14. $f(x) = x^3 + 4x^2 - 11x + 6$
15. $f(x) = x^3 - 7x^2 + x - 7$
16. $f(x) = -2x^3 + 19x^2 - 49x + 20$
17. $f(x) = -17x^3 + 5x^2 + 34x - 10$
18. $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$
19. $f(x) = 3x^3 + 3x^2 - 11x - 10$
20. $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

21. $f(x) = 9x^3 - 5x^2 - x$
22. $f(x) = 6x^4 - 5x^3 - 9x^2$
23. $f(x) = x^4 + 2x^2 - 15$
24. $f(x) = x^4 - 9x^2 + 14$
25. $f(x) = 3x^4 - 14x^2 - 5$
26. $f(x) = 2x^4 - 7x^2 + 6$
27. $f(x) = x^6 - 3x^3 - 10$
28. $f(x) = 2x^6 - 9x^3 + 10$
29. $f(x) = x^5 - 2x^4 - 4x + 8$
30. $f(x) = 2x^5 + 3x^4 - 18x - 27$

In Exercises 31 - 33, use your calculator,[9] to help you find the real zeros of the polynomial. State the multiplicity of each real zero.

31. $f(x) = x^5 - 60x^3 - 80x^2 + 960x + 2304$
32. $f(x) = 25x^5 - 105x^4 + 174x^3 - 142x^2 + 57x - 9$
33. $f(x) = 90x^4 - 399x^3 + 622x^2 - 399x + 90$
34. Find the real zeros of $f(x) = x^3 - \frac{1}{12}x^2 - \frac{7}{72}x + \frac{1}{72}$ by first finding a polynomial $q(x)$ with integer coefficients such that $q(x) = N \cdot f(x)$ for some integer N. (Recall that the Rational Zeros Theorem required the polynomial in question to have integer coefficients.) Show that f and q have the same real zeros.

In Exercises 35 - 44, find the real solutions of the polynomial equation. (See Example 3.3.7.)

35. $9x^3 = 5x^2 + x$
36. $9x^2 + 5x^3 = 6x^4$
37. $x^3 + 6 = 2x^2 + 5x$
38. $x^4 + 2x^3 = 12x^2 + 40x + 32$
39. $x^3 - 7x^2 = 7 - x$
40. $2x^3 = 19x^2 - 49x + 20$
41. $x^3 + x^2 = \dfrac{11x + 10}{3}$
42. $x^4 + 2x^2 = 15$
43. $14x^2 + 5 = 3x^4$
44. $2x^5 + 3x^4 = 18x + 27$

[9] You *can* do these without your calculator, but it may test your mettle!

In Exercises 45 - 54, solve the polynomial inequality and state your answer using interval notation. For help with these exercises, click on the resources below:

- Solving Polynomial Inequalities

45. $-2x^3 + 19x^2 - 49x + 20 > 0$
46. $x^4 - 9x^2 \leq 4x - 12$
47. $(x-1)^2 \geq 4$
48. $4x^3 \geq 3x + 1$
49. $x^4 \leq 16 + 4x - x^3$
50. $3x^2 + 2x < x^4$
51. $\dfrac{x^3 + 2x^2}{2} < x + 2$
52. $\dfrac{x^3 + 20x}{8} \geq x^2 + 2$
53. $2x^4 > 5x^2 + 3$
54. $x^6 + x^3 \geq 6$
55. In Example 3.1.3 in Section 3.1, a box with no top is constructed from a 10 inch $\times$ 12 inch piece of cardboard by cutting out congruent squares from each corner of the cardboard and then folding the resulting tabs. We determined the volume of that box (in cubic inches) is given by $V(x) = 4x^3 - 44x^2 + 120x$, where x denotes the length of the side of the square which is removed from each corner (in inches), $0 < x < 5$. Solve the inequality $V(x) \geq 80$ analytically and interpret your answer in the context of that example.
56. From Exercise 32 in Section 3.1, $C(x) = .03x^3 - 4.5x^2 + 225x + 250$, for $x \geq 0$ models the cost, in dollars, to produce x PortaBoy game systems. If the production budget is \$5000, find the number of game systems which can be produced and still remain under budget.
57. Let $f(x) = 5x^7 - 33x^6 + 3x^5 - 71x^4 - 597x^3 + 2097x^2 - 1971x + 567$. With the help of your classmates, find the x- and y- intercepts of the graph of f. Find the intervals on which the function is increasing, the intervals on which it is decreasing and the local extrema. Sketch the graph of f, using more than one picture if necessary to show all of the important features of the graph.
58. With the help of your classmates, create a list of five polynomials with different degrees whose real zeros cannot be found using any of the techniques in this section.

Checkpoint Quiz 3.3

1. Let $p(x) = 3x^5 + 3x^4 - 6x^3 - 15x^2 - 15x - 6$.

   (a) Use the Rational Zeros Theorem to generate a list of potential rational zeros.

   (b) Use Descartes' Rule of Signs to find the possible number of positive and negative real zeros.

   (c) Find all real zeros of p and state their multiplicities.

   (d) Sketch the graph of $y = p(x)$. Be sure find:

      i. the y-intercept

      ii. the end behavior

      iii. the x-intercepts

2. Solve: $2x^3 + 3 \geq x^2 + 4x$.

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2

3.3.4 Answers

1. For $f(x) = x^3 - 2x^2 - 5x + 6$
   - All of the real zeros lie in the interval $[-7, 7]$
   - Possible rational zeros are ± 1, ± 2, ± 3, ± 6
   - There are 2 or 0 positive real zeros; there is 1 negative real zero
2. For $f(x) = x^4 + 2x^3 - 12x^2 - 40x - 32$
   - All of the real zeros lie in the interval $[-41, 41]$
   - Possible rational zeros are ± 1, ± 2, ± 4, ± 8, ± 16, ± 32
   - There is 1 positive real zero; there are 3 or 1 negative real zeros
3. For $f(x) = x^4 - 9x^2 - 4x + 12$
   - All of the real zeros lie in the interval $[-13, 13]$
   - Possible rational zeros are ± 1, ± 2, ± 3, ± 4, ± 6, ± 12
   - There are 2 or 0 positive real zeros; there are 2 or 0 negative real zeros
4. For $f(x) = x^3 + 4x^2 - 11x + 6$
   - All of the real zeros lie in the interval $[-12, 12]$
   - Possible rational zeros are ± 1, ± 2, ± 3, ± 6
   - There are 2 or 0 positive real zeros; there is 1 negative real zero
5. For $f(x) = x^3 - 7x^2 + x - 7$
   - All of the real zeros lie in the interval $[-8, 8]$
   - Possible rational zeros are ± 1, ± 7
   - There are 3 or 1 positive real zeros; there are no negative real zeros
6. For $f(x) = -2x^3 + 19x^2 - 49x + 20$
   - All of the real zeros lie in the interval $\left[-\frac{51}{2}, \frac{51}{2}\right]$
   - Possible rational zeros are $\pm\frac{1}{2}$, ± 1, ± 2, $\pm\frac{5}{2}$, ± 4, ± 5, ± 10, ± 20
   - There are 3 or 1 positive real zeros; there are no negative real zeros
7. For $f(x) = -17x^3 + 5x^2 + 34x - 10$
   - All of the real zeros lie in the interval $[-3, 3]$
   - Possible rational zeros are $\pm\frac{1}{17}$, $\pm\frac{2}{17}$, $\pm\frac{5}{17}$, $\pm\frac{10}{17}$, ± 1, ± 2, ± 5, ± 10
   - There are 2 or 0 positive real zeros; there is 1 negative real zero

8. For $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$
   - All of the real zeros lie in the interval $\left[-\frac{4}{3}, \frac{4}{3}\right]$
   - Possible rational zeros are $\pm\frac{1}{36}$, $\pm\frac{1}{18}$, $\pm\frac{1}{12}$, $\pm\frac{1}{9}$, $\pm\frac{1}{6}$, $\pm\frac{1}{4}$, $\pm\frac{1}{3}$, $\pm\frac{1}{2}$, ± 1
   - There are 2 or 0 positive real zeros; there are 2 or 0 negative real zeros
9. For $f(x) = 3x^3 + 3x^2 - 11x - 10$
   - All of the real zeros lie in the interval $\left[-\frac{14}{3}, \frac{14}{3}\right]$
   - Possible rational zeros are $\pm\frac{1}{3}$, $\pm\frac{2}{3}$, $\pm\frac{5}{3}$, $\pm\frac{10}{3}$, ± 1, ± 2, ± 5, ± 10
   - There is 1 positive real zero; there are 2 or 0 negative real zeros
10. For $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$
    - All of the real zeros lie in the interval $\left[-\frac{9}{2}, \frac{9}{2}\right]$
    - Possible rational zeros are $\pm\frac{1}{2}$, ± 1, $\pm\frac{3}{2}$, ± 3
    - There are 2 or 0 positive real zeros; there are 2 or 0 negative real zeros
11. $f(x) = x^3 - 2x^2 - 5x + 6$
    $x = -2$, $x = 1$, $x = 3$ (each has mult. 1)
12. $f(x) = x^4 + 2x^3 - 12x^2 - 40x - 32$
    $x = -2$ (mult. 3), $x = 4$ (mult. 1)
13. $f(x) = x^4 - 9x^2 - 4x + 12$
    $x = -2$ (mult. 2), $x = 1$ (mult. 1), $x = 3$ (mult. 1)
14. $f(x) = x^3 + 4x^2 - 11x + 6$
    $x = -6$ (mult. 1), $x = 1$ (mult. 2)
15. $f(x) = x^3 - 7x^2 + x - 7$
    $x = 7$ (mult. 1)
16. $f(x) = -2x^3 + 19x^2 - 49x + 20$
    $x = \frac{1}{2}$, $x = 4$, $x = 5$ (each has mult. 1)
17. $f(x) = -17x^3 + 5x^2 + 34x - 10$
    $x = \frac{5}{17}$, $x = \pm\sqrt{2}$ (each has mult. 1)
18. $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$
    $x = \frac{1}{2}$ (mult. 2), $x = -\frac{1}{3}$ (mult. 2)
19. $f(x) = 3x^3 + 3x^2 - 11x - 10$
    $x = -2$, $x = \frac{3\pm\sqrt{69}}{6}$ (each has mult. 1)

20. $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$
    $x = -1$, $x = \frac{1}{2}$, $x = \pm\sqrt{3}$ (each mult. 1)

21. $f(x) = 9x^3 - 5x^2 - x$
    $x = 0$, $x = \frac{5\pm\sqrt{61}}{18}$ (each has mult. 1)

22. $f(x) = 6x^4 - 5x^3 - 9x^2$
    $x = 0$ (mult. 2), $x = \frac{5\pm\sqrt{241}}{12}$ (each has mult. 1)

23. $f(x) = x^4 + 2x^2 - 15$
    $x = \pm\sqrt{3}$ (each has mult. 1)

24. $f(x) = x^4 - 9x^2 + 14$
    $x = \pm\sqrt{2}$, $x = \pm\sqrt{7}$ (each has mult. 1)

25. $f(x) = 3x^4 - 14x^2 - 5$
    $x = \pm\sqrt{5}$ (each has mult. 1)

26. $f(x) = 2x^4 - 7x^2 + 6$
    $x = \pm\frac{\sqrt{6}}{2}$, $x = \pm\sqrt{2}$ (each has mult. 1)

27. $f(x) = x^6 - 3x^3 - 10$
    $x = \sqrt[3]{-2} = -\sqrt[3]{2}$, $x = \sqrt[3]{5}$ (each has mult. 1)

28. $f(x) = 2x^6 - 9x^3 + 10$
    $x = \frac{\sqrt[3]{20}}{2}$, $x = \sqrt[3]{2}$ (each has mult. 1)

29. $f(x) = x^5 - 2x^4 - 4x + 8$
    $x = 2$, $x = \pm\sqrt{2}$ (each has mult. 1)

30. $f(x) = 2x^5 + 3x^4 - 18x - 27$
    $x = -\frac{3}{2}$, $x = \pm\sqrt{3}$ (each has mult. 1)

31. $f(x) = x^5 - 60x^3 - 80x^2 + 960x + 2304$
    $x = -4$ (mult. 3), $x = 6$ (mult. 2)

32. $f(x) = 25x^5 - 105x^4 + 174x^3 - 142x^2 + 57x - 9$
    $x = \frac{3}{5}$ (mult. 2), $x = 1$ (mult. 3)

33. $f(x) = 90x^4 - 399x^3 + 622x^2 - 399x + 90$
    $x = \frac{2}{3}$, $x = \frac{3}{2}$, $x = \frac{5}{3}$, $x = \frac{3}{5}$ (each has mult. 1)

34. We choose $q(x) = 72x^3 - 6x^2 - 7x + 1 = 72 \cdot f(x)$. Clearly $f(x) = 0$ if and only if $q(x) = 0$ so they have the same real zeros. In this case, $x = -\frac{1}{3}$, $x = \frac{1}{6}$ and $x = \frac{1}{4}$ are the real zeros of both f and q.

35. $x = 0, \frac{5 \pm \sqrt{61}}{18}$

36. $x = 0, \frac{5 \pm \sqrt{241}}{12}$

37. $x = -2, 1, 3$

38. $x = -2, 4$

39. $x = 7$

40. $x = \frac{1}{2}, 4, 5$

41. $x = -2, \frac{3 \pm \sqrt{69}}{6}$

42. $x = \pm\sqrt{3}$

43. $x = \pm\sqrt{5}$

44. $x = -\frac{3}{2}, \pm\sqrt{3}$

45. $(-\infty, \frac{1}{2}) \cup (4, 5)$

46. $\{-2\} \cup [1, 3]$

47. $(-\infty, -1] \cup [3, \infty)$

48. $\left\{-\dfrac{1}{2}\right\} \cup [1, \infty)$

49. $[-2, 2]$

50. $(-\infty, -1) \cup (-1, 0) \cup (2, \infty)$

51. $(-\infty, -2) \cup \left(-\sqrt{2}, \sqrt{2}\right)$

52. $\{2\} \cup [4, \infty)$

53. $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

54. $(-\infty, -\sqrt[3]{3}\,) \cup (\sqrt[3]{2}, \infty)$

55. $V(x) \geq 80$ on $[1, 5-\sqrt{5}] \cup [5+\sqrt{5}, \infty)$. Only the portion $[1, 5-\sqrt{5}]$ lies in the applied domain, however. In the context of the problem, this says for the volume of the box to be at least 80 cubic inches, the square removed from each corner needs to have a side length of at least 1 inch, but no more than $5 - \sqrt{5} \approx 2.76$ inches.

56. $C(x) \leq 5000$ on (approximately) $(-\infty, 82.18]$. The portion of this which lies in the applied domain is $(0, 82.18]$. Since x represents the number of game systems, we check $C(82) = 4983.04$ and $C(83) = 5078.11$, so to remain within the production budget, anywhere between 1 and 82 game systems can be produced.